Tropical lower bounds for extended formulations

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Extended formulations: A brief overview of history
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2014. Lee, Raghavendra, Steurer show that the TSP polytope cannot be described with a semidefinite program of polynomial size.
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Assume we have a system of inequalities

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\begin{align*}
    a_{11}x_1 + \ldots + a_{n1}x_n & \geq b_1, \\
    \vdots \\
    a_{m1}x_1 + \ldots + a_{nm}x_n & \geq b_m.
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We want to construct new inequalities in variables \( x_1, \ldots, x_n, y_1, \ldots, y_k \) such that the above system is equivalent to the following:
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    \exists y_1 \ldots \exists y_k \quad \left\{ 
        c_{11}x_1 + \ldots + c_{n1}x_n + d_{11}y_1 + \ldots + d_{1k}y_k & \geq e_1, \\
        \vdots & \\
        \vdots & \\
        c_{r1}x_1 + \ldots + c_{r1}x_n + d_{r1}y_1 + \ldots + d_{r1}y_k & \geq e_r,
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Why do we care? Because it is easier to optimize linear functionals on polytopes with small number of facets.
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Example: The polytope \( \{|x_1| + \ldots + |x_n| \leq 1\} \) has \( 2^n \) facets but admits an extended formulation of size \( 2n \).
The smallest non-trivial example: Hexagon
Example: Hexagon
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Gouveia, Parrilo, Thomas (2013):

Almost all hexagons have extension complexity six.
Motivating problem

Every convex $n$-gon is a projection of a polytope with at most $k$ facets.
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Denote by $\text{wcc}(n)$ the **worst-case extension complexity** of a convex $n$-gon. That is, $\text{wcc}(n)$ is the smallest $k$ for which the above statement is true.
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These lower bounds are achieved on generic (that is, random) polygons. Can we do better? No answer is known, except for very small $n$. 
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Partial solution:

Shitov (2014): $wcc(n) < 6n/7 + 1$ and $wcc(n) \in o(n)$. 
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Assume a polytope $P$ has vertices $v_1, \ldots, v_n$ and facets $f_1, \ldots, f_m$.

The slack matrix of $P$ is the $m$-by-$n$ matrix whose $(i,j)$-th entry equals the distance from $f_i$ to $v_j$. 
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Theorem (Yannakakis, 1991). The extension complexity of a polytope equals the nonnegative rank of its slack matrix.
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Shitov (2014): Yannakakis’ result holds over any real closed field.
Field of generalized Puiseux series
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Denote by $\mathbb{R}\{\{t\}\}$ the set of all formal sums

$$a = \sum_{e \in E} a_et^e,$$

where $a_e$ are nonzero real numbers and $E \subset \mathbb{R}$ is a well-ordered subset.
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Shitov (2014): Let $\mathbb{R}$ be a real closed field and $P \subset \mathbb{R}^n$ a convex polytope. Then, there is a real polytope with the same extension complexity and combinatorial structure as those of $P$. 
Tropical lower bounds
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Let $T$ be a tropical matrix. The Barvinok rank of $T$ is the smallest possible $k$ such that the equality $T = U \circ V$ holds for $m$-by-$k$ tropical matrix $B$ and $k$-by-$n$ tropical matrix $C$. 
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This theorem can be thought of as a generalization of the Boolean rank bound for the extension complexity. Let’s see how it works...
Example: Another hexagon
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The tropicalized slack matrix equals

\[
\begin{pmatrix}
\infty & \infty & a & 0 & 0 & c \\
0 & \infty & \infty & 0 & 0 & 0 \\
0 & a & \infty & \infty & b & 0 \\
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whose Barvinok rank is 6. Therefore, this hexagon has extension complexity 6.
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Is $wcc(n) \leq \left\lfloor \frac{n + 6}{2} \right\rfloor$?
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In contrast to a computer program, the tropical techniques allow us to study polytopes with some infinitesimally small parameters. There may exist tropical counterexamples to above conjectures...
New result

Vandaele, Gillis, Glineur, Tuyttens (2014) undertake a computational experiment. They propose a conjecture consistent with their data:

Is $wcc(n) \leq [(n + 6)/2]$?

They ask: Do generic convex $n$-gons have the same extension complexity?

A conjecture by Padrol (2015): Is $wcc(n) = \lceil 2\sqrt{2n - 2} - 1 \rceil$?

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Indeed, they exist. The answer is everywhere no: $wcc(9) = 8$. 
An enneagon $E$ such that $xc(E') = 8$
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Thank you!